* S - 2 6 3 5 / 3 0 0 *

S-2635

M. Sc. I (Sem. I) Examination

March / April – 2011

Mathematics

(Ordinary Differential Equations)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fillup strictly the details of ☛ signs on your answer book.	Seat No. :
Name of the Examination : ☛ M. SC. I (SEM. I)	
Name of the Subject : ☛ MATHEMATICS	
☛ Subject Code No. : 2 6 3 5 ☛ Section No. (1, 2,.....) : NIL	Student's Signature

(2) Attempt all questions.

(3) Follow the usual notations and conventions.

(4) Figures on the **right** indicate full marks.

1 Attempt any **two** : **14**

(1) If $\frac{\partial g}{\partial u}$ is continuous in D, then prove that there exists a positive constant K such that $\left|g(t,u,)-g(t,u_2)\right| \le k\,|u_1-u_2|, \ \ \forall (t,u_1),(t,u_2)\in D$ Where D: $\left|t-t_0\right|\le h \quad \left|u-u_0\right|\le b.$

(2) Prove that all the solutions of $x' = A(t)x$ where A(t) is an $n\times n$ continuous matrix on $[0,\infty]$ and x is an n-vector, are stable if they are bounded.

(3) (a) Express $u''+u = v'\sin t \quad v''+v = u'\cos t$. In the vector matrix form.

(b) Show that $g(t,u) = 2u^2\cos^2 t + |u|\sin t$ satisfies the lipschitz condition in the region $R: |t|\le \pi, \ \ |u|\le 1$.

2 Attempt any **two** 14

(1) If $g(t,u)$ is a continuous function of t and u in a closed bounded region $R(a,b)$ and satisfies the lipschitz condition in R then prove that there exists a solution u(t) to the initial value problem $u' = g(t,u)$ with $u(t_0) = u_0$ defined on the interval.

$J: |t-t_0| \le h,\ R(a,b): |t-t_0| \le a, |u-u_0| \le b.$

(2) Prove that the solution $u(t,t_0,u_0) = u_0 \exp[-\alpha(t,t_0)]$ of $u' = -\alpha u, \alpha > 0,$ is asymptotically stable but not strongly stable.

(3) Use the method of successive approximation to solve the intergal equation $u(t) = 1 + \int_0^t (t-s)u(s)ds,\ u_0(t) = 1.$

verify the answer to exact solution.

3 Attempt any **two** : 14

(1) If g is continuous and bounded in some domain D_1 of (t,u) and $u' = g(t,u)$ has a solution $u(t)$ on the interval I through the point $u(t_0,u_0) \in D_1,\ t_0 \in I$ and $(r_2, u(r_2 - 0) \in D_1$ then prove that the solution u(t) of $u' = g(t,u)$ with $u(t_0) = u_0$ extisting on (r_1,r_2) can be continued over interval $(r_1, r_2 + \beta)$ for some $\beta > 0$.

(2) Let $f(t)$ be a continuous function and $\vartheta(t)$ a non-negative continuous function on the interval $t_0 \le t \le t_0 + a$. If a continous function $u(t)$ has the property $u(t) \le f(t) + \int_{t_0}^{t} u(s)\vartheta(s)ds$ for $t \in [t_o, t_o + a]$. then prove that

$u(t) \le f(t) + \int_{t_0}^{t} f(s)\vartheta(s)\exp[\int_s^t \vartheta(T)dT]ds$. for $t \in [t_0, t_0 + a]$.

(3) If the function $g(t,u)$ is continuous for all $t \geq 0, u \geq 0,$ show that $\phi(t)$ is a solution of, $u'' + \mu^2 u = g(t,u),\ u(0) = 0, u'(0) = 1,$ Where $\mu > 0$ for $t \geq 0$ if and only if $\phi(t)$ is a solution of the intergal equation.

$$u(t) = \frac{\sin \mu t}{\mu} + \int_0^t \frac{\sin \mu(t-s)}{\mu} g(s), u(s) ds \ for\ t \geq o$$

4 Attempt any **two**. **14**

(1) If Φ is a fundamental matrix of $x' = A(t)x$ then prove that ψ is a fundamental matrix of its adjoint system $x' = -A^T(t)x$ if and only if $\psi^T \Phi = c$ where c is a constant non-singular matrix.

(2) Find the non-singular matrix T and constuct $T^{-1}AT$ for the matrix $A = \begin{bmatrix} -1 & 1 & -1 \\ 0 & -2 & -9 \\ 0 & 1 & -2 \end{bmatrix}$.

(3) If all the characterstic roots of the characterstic polynomial $L(\lambda)\ of\ L(D)y = 0$ have negative real parts then given any solution $y(t)$ of $L(D)y = 0$ prove that there exists positive numbers $\alpha\ and\ M$ such that $|y(t)| \leq Me^{-\alpha t}, t \geq 0$ and hence $\lim_{t \to \alpha} |y(t)| = 0$ .Where $L(d)y = y^{(n)} + a_1 y^{(n-1)} + ----+ a_n$.

5 Attempt any **two**. **14**

(1) If ϕ is a fundamental matrix of $x' = A(t)x$ then prove that ψ is a fundamental matrix of $x' = A(t)x$ where $\psi(t) = \Phi(t+w); -\infty < t < \infty$ correspoinding to every Φ such that a periodic non-singular matrix P with period w and constant matrix R such that $\Phi(t) = P(t)e^{tR}$.

(2) Solve the third order differential equation $y''' - 3y' + 2y = 9e^t; t \geq o$ with initial condition $y(0) = 0$.

(3) Prove that if all the characterstic roots of A have negative real parts then for any solution $x(t)\ of\ x' = Ax \lim_{x \to \infty} \|x(t)\| = 0$.
